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POPULATION PROJECTIONS FOR AIDS USING AN ACTUARIAL MODEL

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This paper gives details of a model for forecasting AIDS, developed for actuarial purposes, but used also for population projections. The model is only appropriate for homosexual transmission, but it is age-specific, and it allows variation in the transition intensities by age, duration in certain states and calendar year.

The differential equations controlling transitions between states are defined, the method of numerical solution is outlined, and the parameters used in five different Bases of projection are given in detail. Numerical results for the population of England and Wales are shown.

1. Introduction

This paper describes a model for forecasting AIDS, which was designed originally for actuarial use in dealing with life assurance companies and pension funds, but which is also applicable to making projections for AIDS in a total population.

The mathematical model was first outlined in Wilkie (1988*a*), and described more fully in Wilkie (1988*b*, *c*). Since those papers were written, certain enhancements have been made that are described in this paper.

Population projections for the United Kingdom have appeared in Wilkie (1987), Daykin (1987*a*), Cox, Working Group Report (1988) and Daykin *et al*. (1988). Results applicable to life assurance and Permanent Health Insurance have appeared in Daykin (1987*b*, 1988) and Daykin *et al*. (1988). Some revised projections, using some of the enhancements referred to above, have appeared in Daykin (1989).

In Appendix 10 of Cox, Working Group Report (1988) forecasts were given for the population of England and Wales up to 1992. Here, we extend the results up to the year 2012. It is not suggested that these forecasts, on any of the bases used, can be reliable for such a long period ahead, but they show the results of the model for such longer periods; these may be of assistance in understanding the consequences of particular assumptions.

Actuaries require that the model should be age-specific, and should take into account normal age-specific mortality as well as the extra sickness and mortality from AIDS. In this respect this model is more elaborate than most others that have been proposed, though it is less elaborate in other directions.

Any practical model must simplify the complexity of the real world. This actuarial model simplifies the reality of AIDS by assuming that the only mode of transmission of HIV infection is through sexual activity among male homosexuals, and that all reported cases in the United Kingdom have been among homosexuals. This simplification appears reasonable because 84% of reported cases in the U.K. have been among homosexuals. However, it ignores the separate transmission dynamics among haemophiliacs, drug users and heterosexual males and females that in due course may come to play a larger part in the total epidemic of HIV infection.

A further simplification is the assumption that all those males described below as being 'at risk' of infection behave in the same manner at any one time, so that the chance of infection depends on the age of the individual at risk and the particular calendar year, but not on any subcategorization according to frequency of sexual contact or frequency of change of sexual partner.

The model treats males of any single age as forming a 'cohort', and tracks their experience independently of any other cohort. The assumption that only those of the same age infect one another is artificial, but, if infections between those of different ages balance out, it can be considered to be an adequate representation of the truth.

2. The model

The members of one cohort at age x may be in any one of the eleven discrete states that are indicated in figure 1. Five of these are live states: 'clear', 'at risk', 'immune', 'positive' and 'sick from AIDS'. Six are dead states; these are kept separate simply to show the live state that someone died in. The dead states are: 'dead from clear', 'dead from at risk', 'dead from immune', 'dead from positive', 'dead from sick (other than from AIDS)' and 'dead from AIDS'. It may not be possible to distinguish the last two categories, but calculated deaths other than from AIDS of those who suffer from AIDS are comparatively trivial.

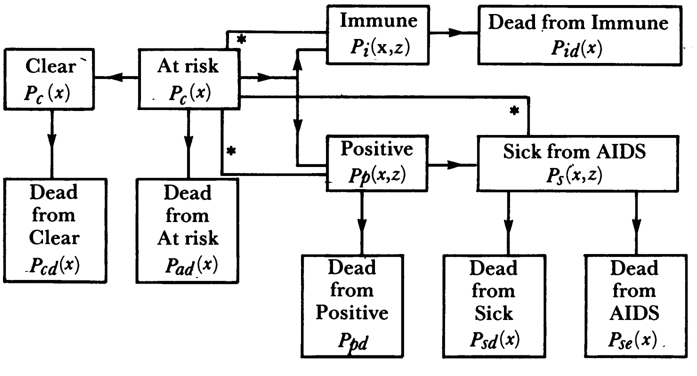

Figure 1 : AIDS Model : States and Transitions
(*denotes possible infection)

Those in the clear state are those whose assumed sexual activity is such that they run no risk whatever of becoming infected with HIV. They form the 'normal' pre-AIDS population for comparative purposes. Those in the at risk state are treated as exposing themselves to the risk of acquiring HIV infection by reason of sexual contact with infected people. Those in the immune state are assumed to have acquired HIV infection and to be infectious, but to be wholly immune from becoming sick from AIDS or dying from AIDS. Whether such a state can exist is not known, but its existence has been suggested by Anderson *et al.* (1986), and it is not difficult to implement this feature of the model.

Those in the positive state are HIV seropositive, but not yet sick from AIDS; they are infectious and not immune. It is assumed that it is possible to distinguish between those who are HIV seropositive and those who are sick from AIDS in a discrete way. In reality there are several stages in the transition from HIV infection to death from AIDS. Those who are suffering from AIDS are thought to be highly infectious, but it is possible that their sexual activity may be considerably reduced. The model makes it possible to choose whether those sick from AIDS are treated as contributing to further infections or not.

It is assumed that the current age (x) is part of the total status, and that transition intensities can all vary by current age. In addition, since each age cohort (or year of birth cohort) is treated separately, each transition intensity can also be varied by calendar year, so that each cohort has its own set of transition intensities.

Duration since entry to the states immune, positive and sick from AIDS are also relevant to the transition intensities. This duration is denoted in each case as z.

Conditional on some initial distribution at a starting age x_0, the probability of an individual being in state j (or the proportion of individuals in state j) at age x is denoted by $p_j(x)$, where $j = [c, a, cd, ad, id, pd, sd, se$ (dead from AIDS)], as in figure 1. The corresponding density for those in the immune, positive or sick from AIDS states at age x and duration z is $p_i(x, z)$, $p_p(x, z)$ or $p_s(x, z)$ respectively, and the total probability of being immune, positive or sick at all durations is denoted $p_i(x)$, $p_p(x)$ or $p_s(x)$.

Possible transitions are as shown in figure 1. Those in any of the live states may die, and those who are sick from AIDS may die from AIDS or from causes other than AIDS. Those who are at risk may change their behaviour and become clear, for example, by giving up sexual activity altogether, or by restricting themselves to one equally monogamous partner. There is no representation in the model of transfer from clear to at risk. Those who are at risk may become infected, and at that point are immediately allocated either to the immune state or to the positive state, in proportions that may depend on age (and on calendar year, though it seems unlikely that this would actually exercise any influence).

Those in the positive state may become sick from AIDS, if they do not die first. Infection is possible from the immunes, positives and sick to the at risk, and how this is represented is described below.

The transition intensity from state j to state k is generally represented by $\mu_{jk}(x)$, if it depends only on age, x, and by $\mu_{jk}(x, z)$ if it depends on both age, x, and duration, z.

3. Differential equations

The usual Kolmogorov differential equations describe the transitions between states, with the exception of the transmission of infection. The differential equations are now described.

$$\mathrm{d}p_c(x)/\mathrm{d}x = -\mu_{cd}(x)\,p_c(x) + \mu_{ac}(x)\,p_a(x); \tag{1}$$

clears are diminished by death and increased by transfers from at risk to clear.

$$\mathrm{d}p_a(x)/\mathrm{d}x = -\mu_{ac}(x)\,p_a(x) - \mu_{ad}(x)\,p_a(x) - T(x); \tag{2}$$

at risk are diminished by transfers to clear and to dead and by the term $T(x)$, representing transfers to the infectious states. This will be described below.

$$\mathrm{d}p_{cd}(x)/\mathrm{d}x = +\mu_{cd}(x)\,p_c(x); \tag{3}$$

dead from clear are increased by deaths among clears. Similar formulae apply to all the dead states.

Put $w = x - z$, so w is the entry age to the given state. Then, for $z > 0$,

$$\mathrm{d}p_i(w+z, z)/\mathrm{d}z = -\mu_{id}(x,z)\, p_i(x,z); \tag{4}$$

immunes at duration z are diminished only by death.

$$\mathrm{d}p_p(w+z, z)/\mathrm{d}z = -\mu_{pd}(x,z)\, p_p(x,z) - \mu_{ps}(x,z)\, p_p(x,z); \tag{5}$$

positives at duration z are diminished by death and by transfers to sick from AIDS.

$$\mathrm{d}p_s(w+z, z)/\mathrm{d}z = -\mu_{sd}(x,z)\cdot p_s(x,z) - \mu_{se}(x,z)\cdot p_s(x,z); \tag{6}$$

those sick from AIDS at duration z may die from causes other than AIDS or may die from AIDS. The new sick at duration 0 are given by,

$$\frac{\partial p_s(x,0)}{\partial x} = \int_0^\infty \mu_{ps}(x,z)\, p_p(x,z)\, \mathrm{d}z. \tag{7}$$

New immunes at duration 0 are given by,

$$\partial p_i(x,0)/\partial x = f_i(x)\; T(x), \tag{8}$$

where $f_i(x)$ is the proportion of those newly infected at age x who are assumed to enter the immune state, and $T(x)$ is the density of newly infected at age x.

New positives at duration 0 are given by

$$\partial p_p(x,0)/\partial x = [1 - f_i(x)]\; T(x), \tag{9}$$

i.e. all those newly infected who do not enter the immune state.

To explain $T(x)$, the newly infecteds, we define terms relating to the total proportions immune, positive and sick at age x as

$$P_i(x) = \int_0^\infty k_i(x,z)\, p_i(x,z)\, \mathrm{d}z, \tag{10}$$

$$P_p(x) = \int_0^\infty k_p(x,z)\, p_p(x,z)\, \mathrm{d}z, \tag{11}$$

and

$$P_s(x) = \int_0^\infty k_s(x,z)\, p_s(x,z)\, \mathrm{d}z. \tag{12}$$

We also define;

$$A_i(x) = \int_0^\infty \mu_{ai}(x,z)\, p_i(x,z)\, \mathrm{d}z, \tag{13}$$

$$A_p(x) = \int_0^\infty \mu_{ap}(x,z)\, p_p(x,z)\, \mathrm{d}z, \tag{14}$$

and

$$A_s(x) = \int_0^\infty \mu_{as}(x,z)\, p_s(x,z)\, \mathrm{d}z. \tag{15}$$

Then

$$T(x) = p_a(x)\left(\frac{A_i(x) + A_p(x) + A_s(x)}{p_a(x) + P_i(x) + P_p(x) + P_s(x)}\right). \tag{16}$$

The intensities of infectivity, $\mu_{ai}(x,z)$, $\mu_{ap}(x,z)$ and $\mu_{as}(x,z)$, are a combination of what other authors have treated as two elements: frequency of sexual contact with a new partner, and

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probability of infection from a new partner. It is assumed that these intensities may vary by the age of the at risks, perhaps representing varying levels of sexual activity at different ages, and may also vary according to how long the partner has been infected.

The terms $k_i(x,z)$, $k_p(x,z)$ and $k_s(x,z)$ allow for the frequency with which immunes, positive and sick respectively enter the pool of sexual exchanges, relative to those at risk who are assumed to enter with unit relative frequency.

If there were no immunes or sick, if the infectivity intensity were a constant μ for all ages and durations, and if $k_p(x,z) = 1$, then the density of new infections would be

$$T(x) = p_a(x)\left(\frac{\mu p_p(x)}{p_a(x)+p_p(x)}\right), \tag{17}$$

which in the absence of any other transitions would lead to the usual logistic growth of the infected proportion in a population.

4. Numerical solution

The technique used is one developed by Waters & Wilkie (1987) that was originally designed to describe sickness with recovery for the purposes of Permanent Health Insurance.

A set of initial conditions is given, and discrete approximations to the differential equations are used to carry forward the initial conditions in steps of length h, where h is some convenient fraction of a year, usually one quarter.

Where duration enters into the status, it is also subdivided into steps of h, i.e. $(0,h)$, $(h,2h)$, etc. The proportions positive, immune or sick with AIDS in each duration step are separately recorded and carried forward from step to step by a discrete approximation. The integrals that appear in equations (10) to (15) are approximated by summations.

Once the initial conditions in a particular starting year and the values of the transition intensities for each future year have been chosen, the forecast is wholly determined.

The model allows a very flexible representation of the transition intensities, varying by age, calendar year and where relevant also by duration in the current state. Our knowledge is too small at this stage to take full advantage of this flexibility.

5. Bases

For the forecasts shown in Appendix 10 of Cox, Working Group Report (1988) five different bases were used, denoted A, B, C, D and E. Although A in general produces the highest and E the lowest numbers, they should not be taken as representing upper and lower limits for the possible course of HIV infection in England and Wales. The reality may turn out to be different from any of these forecasts.

Although the possibility of representing an immune state exists in the model, it was not used in any of the five bases described. Those sick with AIDS were assumed not to contribute to new infections, so that infection was only possible from the positives, who contribute to the denominator of equation (16) to the same extent as the at risk, i.e. $k_p(x,z)$ was taken as unity for all x and z and $k_s(x,z)$ was taken as zero.

The five bases have many assumptions in common and these are described first; the differences are described later.

5.1. *Base Date*

Each projection commences at the end of 1983, and projects forward at annual intervals.

5.2. *Initial Population*

All five start with the estimated population of males aged 15–70 in England and Wales as at mid-1983 taken from 'Population Projections 1983–2023' (Office of Population Censuses and Surveys series PP2 No. 13; HMSO 1985), treated as if this population applied at the end of 1983.

5.3. *New Entrants*

All Bases assume that in each future year the number of males entering the 'population' at age 15 each year is the same as that projected in 'Population Projections 1983–2023'.

5.4. *Mortality other than from AIDS*

All Bases assume that the mortality other than from AIDS of those in the clear, at risk, positive and sick from AIDS statuses is the same as that experienced by the England and Wales male population in 1983, graduated according to the formula,

$$\mu_{cd}(x) = a_0 + a_1 t + \exp\left[b_0 + b_1 t + b_2(2t^2 - 1)\right], \tag{19}$$

where $t = (x-70)/50$, a convenient transformation of age; $a_0 = -0.000780$; $a_1 = -0.001446$; $b_0 = -3.735111$; $b_1 = 4.725108$; $b_2 = -0.662952$. This was derived by graduating the available data using the methods described by Forfar *et al.* (1988).

5.5. *Intensity of developing AIDS (incubation rate)*

All Bases assume that the intensity of developing AIDS is given by a Gompertz formula ($\exp(-8.4+1.4d)$) with a maximum of 0.25, where d is the duration in years since infection.

TABLE 1. PERCENTAGE OF HIV POSITIVE DEVELOPING AIDS AFTER DURATION SHOWN ACCORDING TO FORMULA SHOWN IN TEXT

duration/years	modified Gompertz	Weibull
1	0.05	0.60
2	0.25	3.08
3	1.05	7.90
4	4.24	15.06
5	16.14	24.26
6	34.69	34.89
7	49.13	46.18
8	60.39	57.33
9	69.15	67.62
10	75.97	76.53
11	81.29	83.78
12	85.43	89.33
13	88.65	93.33
14	91.16	96.05
15	93.16	97.78
16	94.64	98.82
17	95.82	99.41
18	96.75	99.72
19	97.47	99.88
20	98.03	99.95

The proportion who have developed AIDS by the given number of years since HIV infection under this assumption are shown in table 1 and can be compared with those from a Weibull distribution with intensity 0.0143 $d^{1.383}$, which are also shown.

5.6. *Initial proportions positive*

For each Basis a particular proportion HIV-positive at each age is assumed in order that approximately the following number of new cases of AIDS in the years 1984 to 1987 are reproduced. In 1984 there were 109 new cases; 224 in 1985; 425 in 1986 and 762 in 1987. In each case it is assumed that there were no sick from AIDS at the end of 1983.

6. Basis A

The assumptions used for basis A are listed below.

6.1. *Initial proportion not clear*

It is assumed that in 1983, 5% of the male population aged 21 to 50 were either at risk or positive. The proportion reduces linearly to 2% at age 15 and to zero at age 70. It is assumed that 2% of new entrants at age 15 in future years are at risk, and that a small fraction of these are positive.

6.2. *Transfer from at risk to clear*

It is assumed that there is no transfer from at risk to clear.

6.3. *Infection intensity*

It is assumed that this has a constant value of 0.7 from ages 25 to 35, reducing linearly outside this range to zero at ages 15 and 70. An infection intensity of 0.7 corresponds to a 'doubling time', in the initial stages of the epidemic, of approximately one year.

6.4. *Mortality of those sick from* AIDS

It is assumed that this is a constant rate of 0.7; this is equivalent to one half of those becoming sick from AIDS dying within one year, one half of the survivors dying within a second year and so on proportionately, giving an average survival time of 1.4 years.

7. Basis B

The assumptions used for basis B are listed below.

7.1. *Initial proportion not clear*

The same as in basis A.

7.2. *Transfer from at risk to clear*

It is assumed that there is a constant transfer from at risk to clear at the rate of 0.1 per year from the beginning of 1987 onwards. This is equivalent to the numbers at risk reducing by 40% every five years in the absence of any other transitions.

7.3. *Infection intensity*

The pattern by age is the same as in basis A, and the rate to the beginning of 1987 is the same, but it is assumed that over the five years from the beginning of 1987 to 1992 the peak infection intensity of 0.7 reduces to 0.35, equivalent to the 'doubling time' itself doubling to two years over this period, and remaining constant thereafter.

7.4. *Mortality of those sick from* AIDS

The same as in basis A.

It can be seen that basis B assumes considerable changes in behaviour as compared with basis A in two ways: first a substantial proportion of those at risk change their behaviour entirely to join the clear status, and those who remain at risk reduce their rate of sexual activity or rate of change of sexual partners to one half of its previous level over the five years from 1987 to 1992. Alternatively this is compatible with an assumption that sexual behaviour is heterogeneous, and those with the highest rate of sexual activity become infected first, with the disease spreading later to those who have a lower rate of sexual activity. However, the model does not specifically represent this pattern.

8. Basis C

The assumptions used for basis C are listed below.

8.1. *Initial proportion not clear*

Half the rates of bases A and B, i.e. 2.5 % from ages 21–50, reducing to 1 % at age 15 and to zero at age 70; new entrants in future years: 1 %.

8.2. *Transfer from at risk to clear*

The same as in basis B.

8.3. *Infection intensity*

The same as in basis B.

8.4. *Mortality of those sick from* AIDS

The same constant rate as in basis A of 0.7 up to the beginning of 1987, reducing linearly over the five years to the beginning of 1992 to a level of 0.35. This represents a lengthening of the survival time of those sick with AIDS resulting from the use of drugs or other treatment. After 1992 the survival rate is such that 30 % of those becoming sick with AIDS die within one year, 30 % of the survivors within a second year and so on, giving an average survival time of 2.8 years.

Basis C makes the same assumptions as basis B about behavioural changes, and in addition assumes half the starting proportions to be not clear, and that Zidovudine (AZT) or other drugs have a significant effect in prolonging the life of those sick with AIDS. It is therefore a fairly 'optimistic' projection, whereas basis A assumes almost no change from the position in the early years (with the exception that those at younger ages and among future new entrants have a lower proportion not clear, i.e. behavioural change comes about in future cohorts rather than among present cohorts).

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9. Basis D

The assumptions used for basis D are the same as those for basis B with two exceptions. The transfer from at risk to clear at a rate of 0.1 per year commences at the beginning of 1984, rather than at the beginning of 1987; and the reduction in the infection intensity takes place over the two years from the beginning of 1984 to the beginning of 1986, rather than from 1987 to 1992.

10. Basis E

The assumptions used for basis E are the same as those used for basis C, with the same two exceptions as differentiate basis D from basis B, that the transfer from at risk to clear commences at the beginning of 1984, not 1987, and that the reduction in infection intensity occurs over the two years from 1984 to 1986.

It can be seen that bases B, C, D and E form a sort of square table of bases, differentiated in pairs.

	5% at risk no AZT	$2\frac{1}{2}$% at risk AZT
behavioural change 1987–92	B	C
behavioural change 1984–86	D	E

11. Results

The results from the projections are shown in tables 2 to 7. The tables show: table 2, projected numbers of deaths from AIDS during each year from 1984 to 2012; table 3, projected numbers of new cases of AIDS during each year from 1984 to 2012; table 4, projected numbers of new HIV infections during each year from 1984 to 2012; table 5, projected numbers of persons currently sick from AIDS at the end of each year from 1983 to 2012; table 6, projected numbers HIV positive, but not sick from AIDS, at the end of each year from 1983 to 2012; table 7, projected total numbers of deaths from AIDS by the end of each year from 1983 to 2012.

It can be seen that basis A (in which no behavioural changes are assumed) produces greater numbers than any of the others. Basis B generally produces greater numbers in the early years than the remaining bases, except that, since those sick from AIDS live much longer in basis C than in basis B, there are more people sick from AIDS at any one time in basis C.

By the end of the century, however, basis D shows a greater number of new HIV infections than basis B, and in due course this would result in a greater number of new cases of AIDS and of deaths from AIDS in basis D than in basis B beyond the final date shown. The reason for this is that a reduction in the infection intensity, which occurs sooner in basis D than in basis B, postpones but does not necessarily eliminate new infections. If the conditions of equation (17) were maintained, then the available population would be 'saturated' in due course, whatever the value of the transition intensity, m. In the full model those in the at risk group who do not change their behaviour to become clear may become infected in the period before they get older when the transition intensities are assumed to reduce to zero. Similarly, basis C produces higher numbers than basis E until about the end of the century.

It is unrealistic to assume that the epidemic of HIV infection will continue with no change whatever in sexual behaviour, so the results from basis A should be taken as indicating a likely upper limit (subject to all the other assumptions being valid). The other bases are more realistic, but there is a very big difference between them. The peak year for numbers of new

Table 2. England and Wales male population: projected numbers of deaths from AIDS during given year

basis... year	A	B	C	D	E
1984	28	28	28	28	28
1985	96	96	96	96	96
1986	208	208	208	206	206
1987	415	415	394	403	383
1988	790	789	693	735	649
1989	1423	1419	1145	1200	985
1990	2477	2452	1784	1726	1310
1991	4241	4131	2635	2266	1571
1992	7078	6642	3986	2854	1942
1993	11305	9881	5905	3536	2490
1994	16961	13393	7864	4322	3059
1995	23545	16520	9535	5191	3630
1996	30035	18693	10680	6099	4170
1997	35282	19724	11234	6982	4640
1998	38518	19799	11277	7765	5003
1999	39557	19161	10926	8373	5234
2000	38678	18010	10294	8750	5320
2001	36391	16522	9479	8865	5266
2002	33253	14852	8566	8724	5089
2003	29760	13126	7618	8357	4812
2004	26293	11434	6686	7814	4464
2005	23092	9841	5802	7152	4072
2006	20267	8386	4990	6427	3662
2007	17835	7090	4259	5686	3254
2008	15762	5960	3616	4969	2863
2009	14002	4991	3059	4301	2500
2010	12506	4174	2582	3700	2173
2011	11239	3494	2179	3173	1882
2012	10172	2933	1842	2719	1630

Table 3. England and Wales male population: projected numbers of new cases of AIDS during given year

basis... year	A	B	C	D	E
1984	110	110	110	110	110
1985	216	216	216	215	215
1986	422	422	421	414	413
1987	811	811	806	771	769
1988	1490	1487	1465	1322	1312
1989	2577	2561	2474	1933	1901
1990	4417	4338	4064	2498	2422
1991	7454	7122	6380	3067	2917
1992	12097	10851	9152	3770	3490
1993	18462	14897	11668	4606	4117
1994	25979	18365	13283	5536	4736
1995	33292	20492	13741	6502	5279
1996	38830	21071	13223	7430	5685
1997	41662	20562	12174	8233	5908
1998	41796	19471	10941	8825	5932
1999	39849	17972	9643	9144	5768
2000	36595	16223	8362	9167	5449
2001	32721	14374	7156	8911	5019
2002	28757	12541	6060	8424	4524
2003	25066	10803	5090	7770	4006
2004	21 838	9208	4249	7017	3495
2005	19114	7782	3534	6226	3016
2006	16840	6535	2934	5446	2582
2007	14932	5464	2438	4711	2200
2008	13318	4560	2032	4045	1873
2009	11943	3808	1702	3459	1597
2010	10772	3188	1435	2955	1368
2011	9783	2682	1220	2529	1179
2012	8962	2270	1048	2174	1025

TABLE 4. ENGLAND AND WALES MALE POPULATION: PROJECTED NUMBERS OF NEW HIV INFECTIONS DURING GIVEN YEAR

basis... year	A	B	C	D	E
1984	4795	4795	4696	4012	3933
1985	8546	8546	8136	4360	4186
1986	14890	14890	13469	4692	4388
1987	24778	23080	19164	5948	5372
1988	38079	29707	21794	7308	6292
1989	52018	32456	20677	8661	7005
1990	61355	30283	16896	9891	7407
1991	62311	24706	12371	10862	7441
1992	55962	20040	9217	11413	7099
1993	46257	17025	7324	11429	6455
1994	36452	13951	5691	10902	5625
1995	28058	11169	4373	9936	4734
1996	21486	8823	3349	8698	3876
1997	16750	6919	2572	7361	3109
1998	13694	5402	1987	6065	2461
1999	11920	4208	1552	4898	1936
2000	10864	3285	1234	3904	1528
2001	10078	2587	1007	3095	1221
2002	9356	2070	846	2462	997
2003	8651	1696	732	1985	838
2004	7985	1429	650	1637	726
2005	7397	1236	588	1388	647
2006	6921	1094	541	1209	588
2007	6574	987	505	1079	544
2008	6363	906	477	982	509
2009	6272	844	457	908	483
2010	6263	797	442	851	462
2011	6291	763	431	806	447
2012	6323	739	425	773	437

TABLE 5. ENGLAND AND WALES MALE POPULATION: PROJECTED NUMBERS SICK FROM AIDS AT END OF GIVEN YEAR

basis... year	A	B	C	D	E
1983	0	0	0	0	0
1984	82	82	82	82	82
1985	202	202	201	201	200
1986	414	414	413	407	407
1987	808	808	823	774	791
1988	1505	1503	1591	1358	1450
1989	2652	2638	2913	2085	2359
1990	4581	4512	5180	2848	3461
1991	7775	7484	8902	3638	4791
1992	12764	11664	14029	4539	6318
1993	19870	16635	19734	5589	7918
1994	28811	21544	25072	6779	9561
1995	38444	25433	29172	8062	11166
1996	47082	27710	31587	9358	12629
1997	53259	28433	32380	10568	13835
1998	56290	27978	31883	11578	14693
1999	56296	26654	30427	12293	15148
2000	53897	24728	28315	12647	15189
2001	49890	22438	25810	12624	14849
2002	45044	19986	23123	12250	14187
2003	39995	17525	20417	11587	13280
2004	35191	15167	17809	10712	12210
2005	30873	12983	15378	9708	11053
2006	27122	11014	13171	8650	9874
2007	23915	9281	11208	7601	8726
2008	21189	7783	9494	6607	7646
2009	18874	6511	8020	5698	6658
2010	16909	5446	6767	4891	5774
2011	15247	4563	5714	4191	4999
2012	13856	3839	4837	3595	4329

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Table 6. England and Wales male population: projected number HIV+ (not sick) at end of given year

basis... year	A	B	C	D	E
1983	6368	6368	6368	6368	6368
1984	11041	11041	10942	10259	10180
1985	19345	19345	18836	14382	14130
1986	33765	33765	31839	18629	18074
1987	57648	55951	50118	23763	22636
1988	94093	84036	70326	29692	27561
1989	143303	113732	88361	36346	32596
1990	199887	139408	100978	43647	37498
1991	254243	156655	106712	51328	41923
1992	297442	165447	106490	58834	45416
1993	324404	167128	101837	65493	47623
1994	333888	162230	93926	70668	48368
1995	327532	152399	84237	73885	47667
1996	308961	139630	74047	74912	45693
1997	282748	125464	64139	73780	42725
1998	253304	110877	54894	70744	39083
1999	224021	96611	46530	66212	35082
2000	196950	83193	39148	60660	30997
2001	173004	70951	32766	54555	27041
2002	152355	60056	27341	48313	23365
2003	134762	50558	22793	42257	20058
2004	119813	42420	19024	36620	17162
2005	107090	35550	15930	31541	14677
2006	96259	29818	13406	27082	12579
2007	87084	25083	11360	23246	10830
2008	79406	21200	9707	19997	9384
2009	73100	18036	8378	17280	8198
2010	68040	15471	7313	15027	7229
2011	64072	13403	6463	13173	6442
2012	61022	11742	5789	11656	5806

Table 7. England and Wales male population: projected total numbers of deaths from AIDS by end of year

basis... year	A	B	C	D	E
1983	0	0	0	0	0
1984	28	28	28	28	28
1985	124	124	124	123	123
1986	332	332	332	329	329
1987	747	747	726	732	713
1988	1537	1536	1419	1467	1361
1989	2960	2954	2563	2667	2347
1990	5436	5406	4347	4393	3657
1991	9677	9537	6982	6659	5228
1992	16755	16179	10968	9513	7170
1993	28060	26060	16873	13049	9660
1994	45021	39454	24738	17372	12718
1995	68567	55973	34273	22563	16348
1996	98601	74667	44954	28662	20517
1997	133883	94391	56188	35643	25157
1998	172401	114190	67465	43408	30160
1999	211958	133351	78391	51782	35394
2000	250636	151361	88685	60531	40714
2001	287028	167883	98164	69396	45980
2002	320281	182735	106730	78120	51069
2003	350042	195861	114348	86477	55881
2004	376335	207295	121033	94290	60346
2005	399427	217137	126836	101442	64418
2006	419694	225523	131825	107869	68080
2007	437528	232613	136085	113555	71334
2008	453291	238573	139701	118524	74196
2009	467292	243564	142759	122825	76697
2010	479798	247739	145341	126525	78869
2011	491037	251233	147520	129698	80752
2012	501210	254165	149362	132417	82381

cases of AIDS ranges from 1995 in basis C to 2000 in basis D, and the numbers of new cases in that peak year range from 21071 in 1996 in basis B to 5932 in 1998 in basis E. The peak years for deaths are a little later and the numbers are similar, whereas the peak years for new infections are much earlier, ranging from 1988 in basis C to 1993 in basis D.

It can be seen how much the results depend on the timing of the change of behaviour of homosexuals. If one could be sure that the change had taken place over 1984–1986, then the projections for the future would be generally lower than if one were to assume that significant changes did not take place until after the widespread publicity campaign during 1986–87. Evidence for the earlier date is discussed in Appendix 5 of Cox, Working Group Report (1988), but this must be taken as circumstantial, rather than conclusive.

It should be noted that the model projects numbers of cases diagnosed, rather than reported. It has been calibrated against estimates of the numbers diagnosed during the years 1984–1987. The number of cases of AIDS reported during 1988 was only 755, but it is not yet clear whether there are significant changes in reporting delays. Because of current delays in reporting, the number of cases diagnosed even for 1987 is not yet known.

TABLE 8. ENGLAND AND WALES MALE POPULATION: PROJECTED AGE-DISTRIBUTION OF NEW CASES OF AIDS

(The numbers in the columns headed A–E are the means of the projected numbers for the end of 1986 and the end of 1987, and the age is the age at becoming sick. The numbers in the column headed compare are the actual numbers for England and Wales reported to CDSC as at 9 March 1988, and the age is the age at diagnosis. The totals may not be the sum of parts because of rounding.)

basis... age	A	B	C	D	E	compare
15–19	3	3	3	2	2	6
20–24	92	92	92	92	92	60
25–29	172	172	172	167	167	144
30–34	196	196	196	190	189	231
35–39	213	213	212	206	206	250
40–44	174	174	174	168	168	185
45–49	126	126	125	124	124	125
50–54	86	86	86	84	84	59
55–59	49	49	49	49	49	30
60–64	29	29	29	29	29	21
65+	10	10	10	10	10	32
total	1150	1150	1150	1124	1122	1143
(plus children and age unknown:						41)

Table 8 shows the projected age-distribution of new cases of AIDS somewhere in the middle of 1987, calculated as the mean of the figures at the end of 1986 and the end of 1987. These are compared with the actual age-distribution of cases of AIDS in England and Wales as reported to CDSC as at 9 March 1988. The distributions match up approximately, though far from exactly.

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